**A24** RECOGNISE AND USE SEQUENCES OF TRIANGULAR, SQUARE AND CUBE NUMBERS, SIMPLE ARITHMETIC PROGRESSIONS, FIBONACCI TYPE SEQUENCES, QUADRATIC SEQUENCES, AND SIMPLE GEOMETRIC PROGRESSIONS (*rn* WHERE *n* IS AN INTEGER, AND *r* IS A RATIONAL NUMBER > 0 **OR A SURD**) **AND OTHER SEQUENCES (higher tier)**

**GEOMETRIC PROGRESSIONS**

In mathematics, a **geometric progression**, also known as a **geometric** **sequence**, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio.

Consider the sequence

2 8 32 128 512

Each term in the sequence is 4 times the previous term.

Consider the sequence

1 −3 9 −27 81

Each term in the sequence is −3 times the previous term.

Sequences such as these are called **geometric progressions**.

Let us write down a general geometric progression, using algebra. We shall take ***a*** to be the first term. The multiplying factor is known as the common ratio and denoted by ***r***.

With this notation, the general geometric progression can be expressed as

*a a × r a × r × r a × r × r × r*

Hence,

*a ar ar*2 *ar*3

**EXAMPLE 1**

Which of these sequences is a geometric progression?

(a) 1 2 3 4 (b) 1 2 4 7

(c) 1 2 4 8 (d) 1 2 3 5

1. 1 2 3 4 No Adding 1 not multiplying by 1

+1

+1

+1

(b) 1 2 4 7 No Not multiplying by the same number each time

+1

+3

+2

(c) 1 2 4 8 Yes Multiplying by 2 each time

×2

×2

×2

(d) 1 2 3 5 No Not multiplying by the same number each time

1 + 2

2 + 3

**EXAMPLE 2**

Find the next two terms of the following geometric sequences.

(a) 1 − 4 16 −64 (b)  2 2 4

1. 1 − 4 16 −64 Multiplier = any term ÷ term before e.g. 16 ÷ −4 = −4

× –4

× –4

× –4

Next term = −64 × −4 = 256 5th term = 4th term × (−4)

Next term = 256 × −4 = −1024 6th term = 5th term × (−4)

(b)  2 2 4 Multiplier = any term ÷ term before e.g. 2 ÷= 2

×

×

×

Next term = 4 × = 4 5th term = 4th term ×

Next term =4 × = 8 4 ×  = 4 × (×) = 4 × 2 = 8

**EXERCISE 1:**

1. Find the next four terms of the following geometric sequences.

(a) 2 6 18 (b) 3 12 48

(c) 5 25 125 (d) 8 12 18

(e) 3 7.5 18.75 (f) 100 50 25

(g) –6 12 –24 (h) –8 2 –0.5

(i) 9 –3 1 (j) 20 4 0.8

2. Find the next three terms of the following geometric sequences.

Leave your answers in surd form.

(a) 2 2 6 (b)  5 5

(c) 5 5 35 (d) 8 4 4

(e) 3  1 (f) 3 6 6

**REAL LIFE PROBLEMS INVOLVING GEOMETRIC SEQUENCES**

The concept of geometric progressions can be applied to real life situations.

**EXAMPLE 3**

Stocks of a company are initially issued at the price of £18.

The value of the stock grows by 20% every year.

(a) Show that the value of a stock follows a geometric sequence.

(b) Calculate the value of the stock ten years after the initial public offering.

(a) It is a geometric sequence Multiplier = 100% + 20% = 120% = 1.20

as multiplied by 1.20 each year

(b) *n*th term = 18 × (1.20)*n*  Starts at £18 and multiplied by 1.20 each year

10th year = 18 × (1.20)10 = 111.4512... 10 years so *n* = 10

New price = £111.45 Put your answer in correct money form to 2 d.p.

**EXAMPLE 4**

A liquid is kept in a barrel.

At the start of a year the barrel is filled with 200 litres of the liquid.

Due to evaporation, at the end of every year the amount of liquid in the barrel is reduced by 12% of its volume at the start of the year.

(a) Calculate the amount of liquid in the barrel at the end of the first year.

(b) Show that the amount of liquid in the barrel at the end of seven years is approximately 81.7 litres.

(a) This is a percentage **decrease** so Multiplier = 100% − 12% = 88% = 0.88

multiplied by 0.88 each year

End of first year= 200 × 0.88 = 176 litres Starts at 200 and multiplied by 0.88 each year

(b) End of 7th year = 200 × (0.88)7 7 years so *n* = 7

= 81.735.... Show extra decimal places for rounding

= 81.7

**EXAMPLE 5**

A tennis ball is dropped from a height of 20 feet.

After the ball hits the floor, it rebounds to 85% of its previous height.

(a) How high will the ball rebound after its third bounce?

(b) Write down a general formula for this sequence.

(a) 85% = 0.85 Multiplier is 85%

After first bounce = 20 × 0.85 = 17 Starts at 20 and multiplier is 0.85

After second bounce = 17 × 0.85 = 14.45 20 × 0.85 × 0.85 = 20 × 0.852 = 14.45

After third bounce = 14.45 × 0.85 = 12.2825 20 × 0.85 × 0.85 × 0.85 = 20 × 0.853   
 = 12.285

(b) *n*th term = 20 × (0.85)*n*

**EXERCISE 2:**

1. A hot tub in a hotel suite is not hot enough.

The hotel tells you that they will increase the temperature by 5% each hour.

If the current temperature of the hot tub is 60°F, what will be the temperature of the hot tub after 10 hours? Give your answer correct to 3 significant figures.

2. A culture of bacteria doubles every 3 hours.

If there are 100 bacteria at the beginning, how many bacteria will there be after 24 hours?

3.   A mine worker discovers an ore sample containing 600 mg of radioactive material.

It is discovered that the radioactive material has a half life of 1 day.

Find the amount of radioactive material in the sample at the end of the 8thday.

4. A fisherman caught 350 kg of fishes on Monday.

From Monday to Friday, the amount of fishes, he caught increased by 10% per day.

What is the total amount of fishes did the fisherman catch in the first five days?

Give your answer as an integer.

5. A city has a population of 30 000 people in the year 2000.

If the population increases 15% per year, what will the population be in 2010?

6. At the age of 16, Tara began receiving yearly payments from a trust fund.

On each succeeding birthday, she received twice as much as in the preceding year.

If she had received a total of £148 617 by age 21, how much did she receive at age 16?

**ANSWERS**

**Exercise 1**

1. (a) 54, 162, 486, 1458 (b) 192, 768, 3072, 12288

(c) 625, 3125, 15625, 78125 (d) 27, 40.5, 60.75, 91.125

(e) 46.875, 117.1875 ,292.96875, 732.421875

(f) 12.5, 6.25, 3.125, 1.5625 (g) 48, – 96, 192, – 384

(h)  (i) 

(j) 

2. (a) 6, 18, 18 (b) 25, 25, 125

(c) 35, 245, 245 (d) 2, 2, 

(e)  (f) 12, 12, 24

**Exercise 2**

1. 97.7°F

2. 25 600

3. 2.34375

4. 2138

5. 121 367

6. £2359